Fermi National Accelerator Laboratory

FERMILAB-PUB-91/20-T
UICHEP-TH/91-2
NUHEP-TH-91-6
UCSD/PTH 91/02

Induced θ Contribution to the Neutron Electric Dipole Moment

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We study the CP violating θ term in QCD induced by the renormalization group evolution of the chromoelectric dipole moments of gluons and heavy quarks. It is pointed out that this renormalization group effect makes the neutron electric dipole moment from the θ term at the hadronic scale dominates over that from other contributions in a large class of models without a Peccei–Quinn symmetry.

Recently, the CP violating three gluon operator $\mathcal{O}_g$ which can be identified as the chromoelectric dipole moment of the gluons has received a lot of attention[1] in connection with its implication for the neutron electric dipole moment (NEDM). It was realized[2,3] that the chromoelectric dipole moment of the b quark, $\mathcal{O}_b^c$, can also play an important role in CP nonconservation. Certainly for any model which gives rise to the standard model as its low energy effective theory, we must include another operator, the topological θ term[4] $\mathcal{O}_\theta$, in considering the NEDM. Even long before the recent developments on the NEDM associated with $\mathcal{O}_g$ and $\mathcal{O}_b^c$, it was noted[5] that $\mathcal{O}_\theta$ can be induced in the course of the renormalization group (RG) evolutions of $\mathcal{O}_g$ and $\mathcal{O}_b^c$. In the previous studies, the effect of this RG induced θ term has been ignored. In this paper, we wish to point out that the NEDM associated with $\mathcal{O}_g$ and $\mathcal{O}_b^c$ is largely dominated by the contribution from the RG induced θ term in models without a Peccei–Quinn symmetry.[6]

To study the induction of $\mathcal{O}_\theta$ due to the renormalization, we first write down the relevent operators defined by

$$\begin{aligned}
\mathcal{O}_g &= (g_s^3/6) f_{abc} G^a_{\mu\alpha} G^{b\alpha}{}_\nu G^c_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma} \quad , \\
\mathcal{O}_b^c &= (g_s/2) G^a_{\mu\nu} \bar{b} i \sigma^{\mu\nu} \gamma_5 (\lambda^a/2) b \quad , \\
\mathcal{O}_\theta &= (g_s^2/64\pi^2) G^a_{\mu\nu} G^a_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma} \quad .
\end{aligned} \tag{1}$$

Of course, this is far from a complete set of CP nonconserving (but flavor conserving since we are interested in the flavor conserving CP-odd quantity the NEDM) operators that can be induced at high energy. Here we made some approximations which can be easily justified in most of the models. First of all we ignored all the operators of dimension 7 or higher. They are typically suppressed by a large mass as dimensional argument required. Secondly, we ignored the four fermion operators because their mixing with $\mathcal{O}_\theta$ is of higher order in both quark masses and the color fine structure constant. Thirdly, we ignored the operators with photon fields because we are only interested in the RG correction due to Quantum Chromodynamics (QCD). Clearly the RG mixing of these operator with $\mathcal{O}_\theta$ is suppressed by the electromagnetic fine structure constant and thus is typically small.

The Weinberg operator $\mathcal{O}_g$ represents the chromoelectric dipole moment of the gluons[7] and its coefficient is known to be suppressed[5,8] by the RG effect when running down to the hadronic scale. Morozov[5] has calculated the mixing among operators in Eq.(1). The renomalization group equations are

$$\begin{aligned}
\mu \frac{d}{d\mu} \mathcal{O}_\theta &= 0 \quad , \\
\mu \frac{d}{d\mu} \mathcal{O}_b^c &= -\frac{2}{3} \frac{\alpha_s}{2\pi} \mathcal{O}_b^c + 4 m_b \mathcal{O}_\theta \quad ,
\end{aligned}$$

$$\mu\frac{d}{d\mu}\mathcal{O}_g = -18\frac{\alpha_s}{2\pi}\mathcal{O}_g - 6\alpha_s^2 m_b \mathcal{O}_b^c \quad . \tag{2}$$

We have checked the entry that was never confirmed by other calculation before. Here the color fine structure constant $\alpha_s(\equiv g_s^2/4\pi)$ and the mass m_b both run according to

$$\mu\frac{d}{d\mu}\alpha_s = -\frac{33-2n}{3}\frac{\alpha_s}{2\pi}\alpha_s, \qquad \mu\frac{d}{d\mu}m_b = -4\frac{\alpha_s}{2\pi}m_b, \tag{3}$$

with n as the number of active flavors at the scale μ.

We have made some approximations in writing down the RG equations above. For example, $\mathcal{O}_\theta$ has a RG mixing with the divergence of a gauge invariant axial vector current of quarks. However since the divergence of a gauge invariant current in the effective Lagrangian does not affect the dynamics, such RG mixing can be ignored. Also through the RG evolution, the chromoelectric dipole moment of a quark will in general induce an imaginary mass for the same quark. This imaginary mass term can also contribute to the induced θ through the chiral anomaly effect when one makes the necessary chiral rotation to keep the quark mass eigenvalues real. However, if one keeps track of the powers of α_s, this effect is actually of higher order in α_s compared to the direct RG mixing with $\mathcal{O}_\theta$. Therefore we have ignored the imginary b quark mass operator induced by $\mathcal{O}_b^c$.

The eigenstates of the RG equation are given by

$$\begin{aligned}
\hat{\mathcal{O}}_g &= \mathcal{O}_g + \frac{36\pi\alpha_s}{7+2n}m_b\mathcal{O}_b^c - \frac{144\pi^2}{5(7+2n)}m_b^2\mathcal{O}_\theta \quad , \\
\hat{\mathcal{O}}_b^c &= \mathcal{O}_b^c - \frac{24\pi}{23-2n}\frac{m_b}{\alpha_s}\mathcal{O}_\theta \quad , \\
\hat{\mathcal{O}}_\theta &= \mathcal{O}_\theta \quad .
\end{aligned} \tag{4}$$

They satisfy,

$$\mu\frac{d}{d\mu}\hat{\mathcal{O}}_g = -\frac{18\alpha_s}{2\pi}\hat{\mathcal{O}}_g \quad , \quad \mu\frac{d}{d\mu}\hat{\mathcal{O}}_b^c = -\frac{2\alpha_s}{6\pi}\hat{\mathcal{O}}_b^c \quad , \quad \mu\frac{d}{d\mu}\hat{\mathcal{O}}_\theta = 0 \quad . \tag{5}$$

Let us now imagine a model for which flavor conserving CP violation can be described by the following effective Lagrangian $\mathcal{L}_{CP}$ at the electroweak scale, say the W boson mass M_W:

$$\mathcal{L}_{CP} = (d_g)_{M_W}\mathcal{O}_g(M_W) + (d_b)_{M_W}\mathcal{O}_b^c(M_W) + (\theta)_{M_W}\mathcal{O}_\theta(M_W) \quad . \tag{6}$$

The above effective Lagrangian can be obtained from a renormalizable model by integrating out heavy particles around (or above) M_W. With the RG equation (2), the effective Lagrangian at the scale m_b^+ just above the b quark threshold is given by,

$$\begin{aligned}
\mathcal{L}_{CP} = &(d_g)_{M_W} K_{W/b}^{54/23} \mathcal{O}_g(m_b^+) \\
&+[(d_b)_{M_W} K_{W/b}^{2/23} + (36\pi/17)(d_g)_{M_W}(m_b\alpha_s)_{m_b}(K_{W/b}^{54/23} - K_{W/b}^{37/23})]\mathcal{O}_b^c(m_b^+) \\
&+[(144\pi^2/1105)(d_g)_{M_W}(m_b^2)_{m_b}(30K_{W/b}^{37/23} - 13K_{W/b}^{54/23} - 17K_{W/b}^{24/23}) \\
&+(d_b)_{M_W}(24\pi/13)(m_b/\alpha_s)_{m_b}(K_{W/b}^{-11/23} - K_{W/b}^{2/23}) + (\theta)_{M_W}]\mathcal{O}_\theta(m_b^+),
\end{aligned} \tag{7}$$

where the renormalization factor $K_{W/b}$ is defined as

$$K_{W/b} = \alpha_s(M_W)/\alpha_s(m_b) \quad . \tag{8}$$

For later convenience, let us define dimensionless quantities $\tilde{d}_g$ and $\tilde{d}_b$ by

$$(d_g)_{M_W} = \tilde{d}_g G_F, \quad (d_b/m_b)_{M_W} = \tilde{d}_b G_F. \tag{9}$$

When removing the b quark from the effective Lagrangian, we use the following substitution[2,3] as a matching condition,

$$(m_b)_{m_b}\mathcal{O}_b^c(m_b^+) \to -\mathcal{O}_g(m_b^-)/32\pi^2 \quad . \tag{10}$$

Then we derive the CP nonconserving interaction at the hadronic scale μ below the charm quark mass,

$$\mathcal{L}_{CP} = [\gamma G_F/\alpha_s(\mu)^2]\mathcal{O}_g(\mu) + [(\theta)_{M_W} + \theta_{\text{ind}}]\mathcal{O}_\theta(\mu) \quad , \tag{11}$$

where

$$\begin{aligned}
\gamma = &\alpha_s(m_c)^2 K_{b/c}^{54/25}\Big\{-(1/32\pi^2)\tilde{d}_b K_{W/b}^{14/23} + \\
&\tilde{d}_g[K_{W/b}^{54/23} - (9/136)(\alpha_s/\pi)_{m_b}(K_{W/b}^{54/23} - K_{W/b}^{37/23})]\Big\} \quad , \\
\theta_{\text{ind}} = &G_F(m_b^2)_{m_b}[(24/13)(\pi/\alpha_s)_{m_b}(K_{W/b}^{1/23} - K_{W/b}^{14/23})\tilde{d}_b \\
&+(144\pi^2/1105)\tilde{d}_g(30K_{W/b}^{37/23} - 13K_{W/b}^{54/23} - 17K_{W/b}^{24/23})] \quad .
\end{aligned} \tag{12}$$

Here we have defined γ so that it does not run, just like θ, when $\mu < m_c$. Numerically, we have

$$\gamma = 1.1 \times 10^{-2}\tilde{d}_g - 10^{-4}\tilde{d}_b, \quad \theta_{\text{ind}} = 2.2 \times 10^{-3}\tilde{d}_b - 3.1 \times 10^{-4}\tilde{d}_g \quad , \tag{13}$$

for the QCD scale $\Lambda = 150$ MeV. The neutron electric dipole moment is related to the parameters in the effective Lagrangian of Eq. (6) as

$$\begin{aligned}
D_n &= e\gamma\xi_g G_F M_\chi[4\pi/g_s(\mu)] + e\theta\xi_\theta M_\chi^{-2} m_u m_d/(m_u + m_d) \\
&= \left(6.7\,\gamma\,\xi_g \times 10^{-19} \times +1.3\,r\theta\,\xi_\theta \times 10^{-16}\right) \mathrm{e\,cm} \quad ,
\end{aligned} \tag{14}$$

where $\theta = (\theta)_{M_W} + \theta_{\rm ind}$, the chiral symmetry breaking scale $M_\chi = 2\pi F_\pi = 1.19$ GeV, $r = (m_d/9\,{\rm MeV})m_u/(m_u + m_d)$, and ξ_g and ξ_θ are coefficients associated with the unknown nonperturbative QCD dynamics. We then have

$$\begin{aligned} D_n = &[(1.3 \times 10^4 (\theta)_{M_W} + 29\tilde{d}_b - 4\tilde{d}_g)\,\xi_\theta\, r \\ &+ (0.74\tilde{d}_g - 0.0067\tilde{d}_b)\,\xi_g] \times 10^{-20}\, e\,{\rm cm}. \end{aligned} \tag{15}$$

We used, for light quark masses[9], $m_u = 5$ MeV and $m_d = 9$ MeV which give $r = 0.36$. The coefficients ξ_g and ξ_θ are chosen such that the estimate using naive dimensional analysis[10] (NDA) gives $|\xi_g| \simeq |\xi_\theta| \simeq 1$. Following Weinberg[1], the renormalization point for the NDA rule is chosen such that $g(\mu) = 4\pi/\sqrt{6}$. However for the NEDM from $\mathcal{O}_\theta(\mu)$, more careful analyses[4] are available and all of them give $|\xi_\theta| \geq 1$. For example using the current algebra technique[11], one finds $|\xi_\theta| \simeq 7.7$. About the NEDM from three gluon operator $\mathcal{O}_g(\mu)$, although as naive as the NDA, a simple scaling argument[12] has been used to obtain $|\xi_g| \simeq 1/30$ which is significantly smaller than the NDA value.

It is interesting to compare the contributions to NEDM from the RG induced θ term to those from $\mathcal{O}_g(\mu)$ at the hadron scale μ. For $r = 0.36$, we have

$$\delta D_n(\mathcal{O}_g \to \mathcal{O}_\theta)/\delta D_n(\mathcal{O}_g \to \mathcal{O}_g) \simeq 2\,(\xi_\theta/\xi_g) \quad , \tag{16}$$

$$\delta D_n(\mathcal{O}_b^c \to \mathcal{O}_\theta)/\delta D_n(\mathcal{O}_b^c \to \mathcal{O}_g) \simeq 1.6 \times 10^3(\xi_\theta/\xi_g) \quad , \tag{17}$$

where the arrows denote the RG evolution from M_W down to the hadronic scale μ. The above result, together with the discussion of ξ_θ and ξ_g in the previous paragraph, implies that the NEDM associated with $\mathcal{O}_b^c(M_W)$ and $\mathcal{O}_g(M_W)$ may be dominated by the RG induced θ term, *instead of* $\mathcal{O}_g(\mu)$ which has been considered in the previous works[2,3], unless a significant cancellation occurs between $(\theta)_W$ and the RG induced $\theta_{\rm ind}$ at the hadronic scale. This is particularly true for $\mathcal{O}_b^c(M_W)$ and can be understood from the fact that the RG induced θ term is not renormalized around the hadron scale while the coefficient of the threshold induced $\mathcal{O}_g$ is strongly suppressed by the subsequent renormalzation effect.

Let us now consider the implication of our analysis for models that incorporate CP violation. We first note that our analysis poses no problem for models with a Peccei–Quinn symmetry since in these models the θ at low energy, $\theta = (\theta)_{M_W} + \theta_{\rm ind}$, can rotated away by a Peccei–Quinn transformation. Thus in what follows, we concentrate on models without a Peccei–Quinn symmetry. Then our analysis implies that the θ term at the hadronic scale μ gives a dominant contribution to the NEDM in models for which flavor conserving CP violation at the electroweak scale can be described by the effective Lagrangian of Eq. (6). Note that the result of Eq. (15) clearly shows that the NEDM is dominated by the contribution from the θ term at the hadronic scale μ over most of the parameter space of coefficients $(\theta)_{M_W}$, $\tilde{d}_g$, and $\tilde{d}_b$, except for a narrow region on which there occurs a dramatic cancellation between $(\theta)_{M_W}$ and $\theta_{\rm ind} = 2.2 \times 10^{-3}\tilde{d}_b - 3.1 \times 10^{-4}\tilde{d}_g$.

The values of $\tilde{d}_g$ and $\tilde{d}_b$ are model-dependent of course. Clearly they are negligibly small in the standard model. However in many interesting extensions of the standard model, their values can be large enough to give the NEDM observable in near future. In such models, $\tilde{d}_g$ arises from two–loop diagrams[1,13,14] at high energy scales around M_W. Its typical size is about

$$(16\pi^2)^{-2} \times |\sin\delta_{CP}| \lesssim 10^{-5} \quad , \tag{18}$$

where δ_{CP} denotes a CP violating phase in the interaction Lagrangian of heavy particles that appear in the two loop diagrams. For the range of $\tilde{d}_g$ given by Eq. (18), both the NEDM from $\mathcal{O}_g(M_W)$ via the RG induced $\mathcal{O}_\theta(\mu)$ and that via $\mathcal{O}_g(\mu)$ would be within the current experimental bound[15] $10^{-25} e\,\mathrm{cm}$ if $\delta_{CP} \lesssim 0.1$.

Unlike $\tilde{d}_g$, the coefficient $\tilde{d}_b$ occurs usually at the one–loop level[2,3] and therefore can be significantly larger. For example, in the left–right symmetric model[2], we have

$$\tilde{d}_b = (4\sqrt{2}\pi^2)^{-1}(m_t/m_b)\sin\xi\sin\eta\; f(m_t^2/M_W^2) \quad , \tag{19}$$

where $\xi(< 0.0055)$[16] is the mixing angle between W_L and W_R and η is the CP violating phase. The function

$$f(h) = \frac{2}{(1-h)^2}[1 + \frac{1}{4}h + \frac{1}{4}h^2 + \frac{3h\log h}{2(1-h)}] \tag{20}$$

is typically of order one. In the multi Higgs doublet Higgs model,[2,3] the charged Higgs boson exchange gives rise to

$$\tilde{d}_b = (4\sqrt{2}\pi^2)^{-1} Im\ Z\ g(m_t^2/M_H^2) \quad . \tag{21}$$

where ImZ is the CP violating parameter and the function

$$g(h) = \frac{h}{(1-h)^2}\Big(h - 3 - \frac{2\log h}{1-h}\Big) \tag{22}$$

is again typically of order one.

For the natural range of $\tilde{d}_b$ given by Eqs. (19,21), the NEDM associated with $\mathcal{O}_b^c(M_W)$ via the threshold induced $\mathcal{O}_g$ is within the experimental bound. However the RG induced θ term will give rise to a too large NEDM in Eq. (15) for natual CP violation, *i.e.* η or $Im\ Z \simeq 0.1 \sim 1$ unless there occurs a dramatic cancellation between $\theta_{\rm ind}$ and $(\theta)_{M_W}$. Then barring this cancellation, the NEDM from the RG induced θ term in Eq. (15) gives the upper bound

$$|\tilde{d}_b| \leq 1.3 \times 10^{-7}, \tag{23}$$

if one uses the current algebra result $|\xi_\theta| = 7.7$. Such bound can be translated into stringent constraints on the CP violating parameter in the left-right symmetric model and the multi Higgs doublet model as

$$\begin{aligned} |\sin\xi\ \sin\eta| &\lesssim 4 \times 10^{-7} \quad ; \\ |Im\ Z| &\lesssim 10^{-5} \quad (\text{for } M_H \simeq m_t). \end{aligned} \tag{24}$$

To conclude, we have argued that the θ term at the hadronic scale gives a dominant contribution to the NEDM in models without a Peccei–Quinn symmetry for which flavor conserving CP violation at the electroweak scale can be described by the effective Lagrangian of Eq. (6). Furthermore barring the cancellation between $(\theta)_{M_W}$ and θ_{ind}, the θ term induced by the RG evolution of the chromoelectric dipole moments of the b quark leads to a strong bound on CP violating parameters in the left-right symmetric model and the multi Higgs doublet model. For most of the models of CP violation without a Peccei Quinn symmetry in the literature, one still has to resort to a fine-tunning of parameters to avoid the strong CP problem. Even in that case, our analysis shows that one has to be very careful about the scale at which the fine-tuning is performed. In that sense, it makes the fine-tuning even less attractive.

We are benefitted from discussions with Dr. T.C. Yuan and Professor Eric Braaten. This work was supported in part by the U.S. Department of Energy and in part by the Research Corporation.

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[1]S. Weinberg, Phys. Rev. Lett. **63**, 2333 (1989).

[2]D. Chang, W.–Y. Keung, C. S. Li, and T. C. Yuan, Phys. Lett. **B241**, 589 (1990).

[3]G. Boyd, A.K. Gupta, S.P. Trivedi, and M.B. Wise, Phys. Lett. **241B**, 884 (1990).

[4]J. E. Kim, Phys. Rep. **150**, 1 (1987); H. Y. Cheng, Phys. Rep. **158**, 1 (1988); R. D. Peccei, DESY Report No. DESY 88-109, 1988, and references cited there.

[5]A. Yu. Morozov, Yad. Fiz. **40**, 788 (1984) [Sov. J. Nucl. Phys. **40**, 505 (1984)].

[6]R. Peccei and H. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); Phys. Rev. **D16**, 1791 (1977).

[7]E. Braaten, C. S. Li, and T. C. Yuan, Phys. Rev. **D42**, 276 (1990).

[8]E. Braaten, C. S. Li, and T. C. Yuan, Phys. Rev. Lett. **64**, 1709 (1990).

[9]J. Gasser and H. Leutwyler, Phys. Rep. 87, 77 (1982). However it has been argued recently that, due to instanton effects, the value of m_u significantly smaller than 5 MeV (even zero) can be consistent with all phenomenological data. For this, see D. Kaplan and A. Manohar, Phys. Rev. Lett. **56**, 2004 (1986); K. Choi, C. W. Kim, and W. K. Sze, Phys. Rev. Lett. **61**, 794 (1988).

[10]A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984); H. Georgi and L. Randall, Nucl. Phys. **B276**, 241 (1986).

[11]R. Crewther, P. DiVecchia, G. Veneziano, and E. Witten, Phys. Lett. **89B**, 123 (1979).

[12]I. I. Bigi and N. G. Uraltsev, "Induced Multi-Gluon Couplings and The Neutron Electric Dipole Moment", UND-HEP-90-BIG02 (1990)

[13]D. A. Dicus, Phys. Rev. **D41**, 999 (1990); J. Dai, H. Dykstra, R. Leigh, S. Paban, and D. Dicus, Phys. Lett. **B237**, 216 (1990).

[14]D. Chang, C.S. Li, and T.C. Yuan, Phys. Rev. D**42**, 867 (1990).

[15]K.F. Smith *et. al.*, Phys. Lett. **B234**, 191 (1990).

[16]D.P. Stoker et.al. *Phys. Rev. Lett.* **54** (1985) 1887; I.L. Bigi and J.M. Frere, *Phys. Lett.* **B110** (1982) 255; J.F. Donoghue and B. Holstein, *Phys. Lett.* **B113** (1982) 383; L. Wolfenstein, *Phys. Rev.* **D29** (1984) 2130.